Entrance Examination - Mathematics

Name and Surname – fill in the field	*Application No.*	*Test Sheet No.*
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Sets, relations, functions, logic

1 Which of the following relations on the set $\{a, b, c\}$ **is not a subset of any order relation** on the set $\{a, b, c\}$? (Order is a reflexive, antisymmetric and transitive relation.)

A $\{(a, a), (b, b)\}$
B $\{(a, a), (b, c), (b, a)\}$
C $\{(a, a), (a, b), (b, c)\}$
***D** $\{(a, b), (b, c), (c, a)\}$
E $\emptyset$ (i.e. empty relation)

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2 Which of the following relations on the set of integers $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ is **transitive**?

A $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x + y = 3\}$
B $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid 2x = y\}$
C $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x = -y\}$
***D** $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x = y\}$
E $\{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x \neq y\}$

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3 Consider the statement **"Every student who had passed the entrance exams was admitted."** Which of the following statements is its negation?

A All students were admitted.
B There exists a student who had not passsed the entrance exams and was admitted.
C No student was admitted.
D No student who had passed the entrance exams was admitted.
***E** There exists a student who had passed the entrance exams and was not admitted.

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4 Let us have an arbitrary set A and an arbitrary function f of type $A \to A$. Which of the following statements is generally true for the function f and every two arbitrary elements $x, y \in A$?

A $f(x) \neq x$.
***B** If $x = y$, then $f(x) = f(y)$.
C $f(x) = x$.
D If $x \neq y$, then $f(x) \neq f(y)$.
E If $f(x) = f(y)$, then $x = y$.

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5 How many **satisfying assignments** does the formula $A \Rightarrow (B \vee (B \Leftrightarrow C))$ have? (A, B and C are distinct propositional variables.)

A 2
B 8
C 1
***D** 7
E 4

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6 For given sets A and B, let $\mathcal{P}(A)$ denote the set of all subsets of the set A and $A \setminus B$ denote the set difference of the sets A and B. How many elements does the set $\mathcal{P}(\{a, b, c\}) \setminus \mathcal{P}(\{a, b\})$ contain?

A 1
***B** 4
C 5
D 6
E 2

Linear algebra

7 Let $f : \mathbb{R} \to \mathbb{R}$ be a linear mapping and let x, y be such that it holds $f(x + y) = 5$ and $f(2x + 3y) = 12$. What is the value of $f(x)$?

***A** 3
B 2
C 0
D 1
E It is not possible to determine the value of $f(x)$ from the given information.

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8 Calculate the determinant of the following matrix:

$$\begin{pmatrix} 3 & -2 & -3 \\ -1 & 5 & 7 \\ 1 & 0 & 0 \end{pmatrix}$$

A 2
B 4
C 5
D 0
***E** 1

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9 $\begin{pmatrix} 2 & 1 & -3 \\ 5 & -2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 \\ 6 & 5 \\ 5 & 3 \end{pmatrix} =$

A The product of given matrices is not well defined.
B $\begin{pmatrix} -3 & 2 & 1 \\ 0 & 1 & 3 \\ 2 & 5 & 1 \end{pmatrix}$
C $\begin{pmatrix} 6 & 2 \\ 30 & -10 \end{pmatrix}$
D $\begin{pmatrix} 5 & 3 \\ 11 & 3 \end{pmatrix}$
***E** $\begin{pmatrix} -3 & 0 \\ -2 & -3 \end{pmatrix}$

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10 Which of the following triples of vectors is linearly independent?

A $(-1,0,1),(2,0,-2),(1,1,1)$
B $(2,0,2),(1,1,0),(6,4,2)$
C $(1,2,3),(3,2,1),(2,2,2)$
***D** $(1,1,1),(1,1,0),(1,0,0)$
E $(3,3,3),(4,4,4),(-2,-2,-2)$

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11 Consider the following system of linear equations over $\mathbb{R}$:

$$\begin{aligned} 3x + 2y + z &= 2 \\ x - 2y + z &= -2 \\ -2x - 4y - 4z &= -4 \end{aligned}$$

Which of the following claims holds?

A The system has infinitely many solutions and the set of all solutions is a line in $\mathbb{R}^3$.
***B** The system has exactly one solution.
C Every point of $\mathbb{R}^3$ is a solution of the system.
D The system has no solution.
E The system has infinitely many solutions and the set of all solutions is a plane in $\mathbb{R}^3$.

Calculus

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12 Compute the value of the integral $\int_0^2 6x^2 - \cos(\pi x)\mathrm{d}x$.

A 24
B 1
***C** 16
D 2π
E 4

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13 Which of the following functions of type $\mathbb{R} \to \mathbb{R}$ is **surjective**?

***A** x^3
B x^2
C $\sin x$
D $|x|$
E $1/x$

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14 Let us have the function $f(x) = 2x^5 + e^{2e} + e^{2x}$. Which of the following functions is equal to the derivative of the function f?

A $5x^4 + e^{2x}$
***B** $10x^4 + 2e^{2x}$
C $5x^4 + e^2$
D $5x^4 + 2e^{2x}$
E $10x^4 + 2e^2$

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15 What is the value of the following series? $\sum_{i=0}^{\infty}(\frac{1}{2^i} - \frac{1}{2^{i+1}})$

A It is not possible to determine the value because the series diverges to $+\infty$.
B 2
C $\frac{1}{2}$
D It is not possible to determine the value because the series oscillates.
***E** 1

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16 We say that a function $f : \mathbb{R} \to \mathbb{R}$ is *even* if $\forall x \in \mathbb{R} : f(-x) = f(x)$ and that f is *odd* if $\forall x \in \mathbb{R} : f(-x) = -f(x)$. Choose the correct statement.

A The function $f(x) = \cos x$ is neither even nor odd.
B The function $f(x) = x^3 - x$ is even.
***C** The function $f(x) = \sin x^2$ is even.
D The function $f(x) = 2\sin x + x$ is both even and odd.
E The function $f(x) = |x|$ is odd.

Probability

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17 Consider a standard six-sided die. Which of the following random events are **stochastically independent**?

A a number smaller than 4 is rolled; number greater than 4 is rolled
***B** an even number is rolled; number 5 or 6 is rolled
C an even number is rolled; 1 is rolled
D an even number is rolled; number 4, 5, or 6 is rolled
E an odd number is rolled; 1 is rolled

18 Let us have a biased coin, that comes up heads with the probability 30 % and tails with the probability 70 %. What is the probability, rounded to whole integers, that exactly 3 out of 5 flips come up heads?

A 2 %
B 1 %
***C** 13 %
D 7 %
E 26 %

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19 Consider a probability space containing two random events A and B. What is the probability of $(A \cup B)$?

A $P(A) + P(B)$
B $P(A) \cdot P(B)$
C $(P(A) \cdot P(B)) + P(A \cap B)$
D $P(A) + P(B) + P(A \cap B)$
***E** $P(A) + P(B) - P(A \cap B)$

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20 Consider a random variable X such that $P(X = 0) = \frac{1}{4}$, $P(X = 2) = \frac{1}{2}$, $P(X = 4) = \frac{1}{4}$. Compute the **variance** of the random variable X. (Here $P(X = y)$ denotes the probability of the random variable X attaining the value y.)

***A** 2
B 6
C 4
D 8
E 10

Graph theory

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21 Which of the following claims is true for every connected undirected graph with 5 vertices, at least two of which are of degree 1?

A Graph has at least 5 edges.
***B** Graph has at most 5 edges.
C Graph contains a vertex of degree 3.
D Graph is a cycle.
E Graph is a tree.

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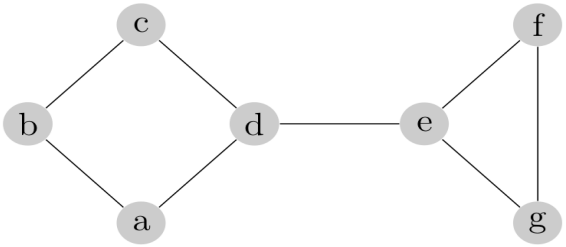
22 Consider the following graph:



How many different spanning trees does it have?

A 14
B 11
C 13
D 10
***E** 12

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23 How many triangles are there in the complete graph with 4 vertices? (Triangle is a graph with 3 vertices in which each pair of vertices is connected by an edge.)

A 2
B 3
C 5
***D** 4
E 6

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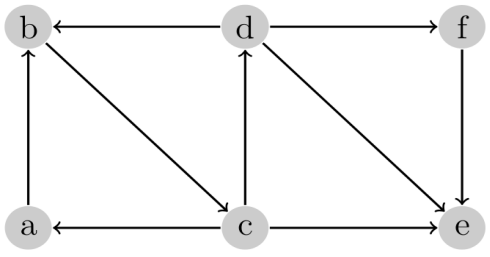
24 Consider the following directed graph:



Decide which of the following claims about **breadth-first search** starting from vertex a holds. (We do not assume any ordering on the out-neighbors, i.e. the order in which breadth-first search algorithm visits the neighbors of a vertex is ambiguous.)

A Vertex f can be discovered before vertex e.
***B** Vertex f will always be the last discovered vertex.
C Vertex e can be the last discovered vertex.
D Vertex e will always be discovered before vertex b.
E Vertex d will always be discovered before vertex e.

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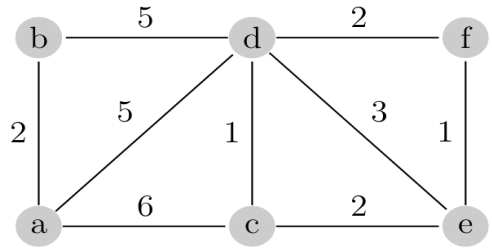
25 Consider the following undirected, edge-weighted graph:



What is the weight (i.e. the sum of edge weights) of its minimal spanning tree?

A 14
B 9
C 15
D 13
***E** 11